### Notes on May 2017 version of the eclips subroutine

The *eclips* routine detects the eclipsing turns and/or shadow crossings and depending on the GNSS satellite type, it rotates the input body-x vector (which is assumed to be oriented according to nominal yaw model) by yaw angles specified by the applicable eclipsing yaw model. The current version Feb. 2017 (eclips Feb2017,f) handles four GNSS', namely GPS, GLONASS, Galileo and Beidou. The subroutine uses the same body-x convention for all GNSS satellites, including the GPS Block IIR ones, i.e., the positive body-x points towards the Sun hemisphere, which is now considered the IGS standard (Montenbruck et al 2015). Within the subroutine, the GNSS satellites are distinguished by PRN number (the input parameter IPRN). IPRN of 1-32 are reserved for GPS, 33-64 for GLONASS, 65-100 for Galileo and 101-136 for Beidou. Additionally, to invoke an appropriate eclipsing model, GPS and Beidou satellites require the input parameter *IBLK* to specify the GPS Block (IBLK = 3 for IIA; 4 or 5 for IIR/IIRM and 6 for IIF), or the Beidou satellite type (IBLK = 21 or 25 for MEO; 22 or 26 for IGEO and 23 or 27 for GEO satellites). GPS and GLONASS also require the input parameter ANIGHT, specifying shadow entry and exit limits, e.g., 180 + 13.25 deg and 180 + 14.2 deg for GPS and GLONASS, respectively. ANIGHT for Galileo and Beidou are specified by the respective eclipsing models, i.e., 180+15 and 180+2 deg., respectively. For GPS and GLONASS, the noon turn limit parameter ANOON (which specify when the nominal yaw rates start to exceed the hardware ones) is computed internally within the subroutine, as it depends on individual satellites, or a satellite block type. The constant ANOON of 15 and 2 deg is used for Galileo and Beidou, respectively (see below for more details on Galileo and Beidou eclipsing models).

There are two main groups of updates/modifications, to the previously released versions of December 2013. The implementation and changes related the Galileo and Beidou eclipsing models are labelled as "*Jan 10, 2017*". The label of "*Feb 27, 2017*" is used to flag all the modifications related to accounting for a possible Sun angle  $\beta$  sign change during eclipsing turns as well as the anomalous noon turns of the Block IIF and IIA satellites with small negative or positive  $\beta$ , respectively. The labelling of all the changes hopefully will simplify updating of already implemented versions of the *eclips* subroutine.

## Beidou eclipsing model (source-code changes/updates "Jan 10, 2017")

The Beidou eclipsing model is fairly simple and straightforward. It depends on the orbit type of Beidou satellites. According to Montenbruck et al (2015), all the Beidou geosynchronous (GEO) satellites always employ the orbit normal (ON) yaw, i.e. the body-x points in the direction of the satellite velocity (Yaw angle = 0 deg), regardless of whether satellites are eclipsing or not. Consequently, within the subroutine, the body-x orientation of Beidou inclined geosynchronous (IGEO) and mean earth orbit (MEO) satellites is unchanged, i.e., it uses the usual nominal yaw model, however, when the Sun angle  $|\beta| \le 4$  deg, the ON yaw is used as well, regardless of whether satellites are in eclipsing turns or not. In the subroutine, the  $\beta$  limit of 4 deg is hard coded (2\**BETA0* = 2). The 2\**BETA0* is also used for the Beidou *ANOON* and *ANIGHT* eclipsing detection limits, which are used only for an eclipse reporting/print out. All the above Beidou eclipsing models. Note that setting *BETA0* = 0 disables eclipsing (the ON yaw) for IGEO and MEO Beidou as well as all the Galileo satellites (see below). It also disables the eclipse reporting for all the Beidou as well as all the Galileo satellites (see below). It also disables the eclipse reporting for all the Beidou as well as all the Galileo satellites (see below).

### Galileo eclipsing model (source-code changes/updates "Jan 10, 2017")

The official Galileo eclipsing model is given in <u>https://www.gsc-europa.eu/support-to-developers/galileo-iov-satellite-metadata</u>, see the *Section 3.1* "*Yaw Steering Law*"), and it is applicable within the eclipsing limits of  $|S_x| < \sin \beta_x$  and  $|S_y| < \sin \beta_y$ , with  $\beta_x = 15 \text{ deg}$ ,  $\beta_y = 2 \text{ deg}$ , where the Sun unit vector is evaluated from orbit angle  $\mu$  (measured from the orbit noon - the orbit point closest to the Sun) and the Sun angle  $\beta$ 

$$(S_{x}, S_{y}, S_{z})^{\mathrm{T}} = (-\sin\mu\cos\beta, -\sin\beta, -\cos\mu\cos\beta)^{\mathrm{T}}.$$
(1)

The second eclipsing limit is exactly equivalent to  $|\beta| < 2 \text{ deg}$ , whereas, since  $|\beta| < 2 \text{ deg}$  and  $\cos\beta \sim 1$ , the first eclipsing limit can be approximated (to within a fraction of a degree) by the orbit angle limits of

 $|\mu| < 15 \text{ deg for shadow, or } |\mu - 180| < 15 \text{ deg for noon turns, respectively.}$  To nearly the same approximation, the satellite-Earth-Sun angle *E* can be used instead of the orbit angle  $\mu$  in the above limits. This is why, consistently with the other GNSS', the regular eclipsing detection, utilizing *E*, with limits of ANOON = 15 and ANIGHT = 180+15 deg has also been adopted here for Galileo.

The Galileo nominal yaw angle  $\psi_n$  (consistent with the IGS body-x and  $\mu$  conventions) is computed as

$$\psi_n = ATAN2\left(\frac{S_y}{\sqrt{1-S_z^2}}, \frac{S_x}{\sqrt{1-S_z^2}}\right), \tag{2}$$

which is exactly equivalent to

$$\psi_n = ATAN2(S_y, S_x). \tag{3}$$

as well as to the usual formulation, used within this subroutine:

$$\psi_n = ATAN2(-\tan\beta,\sin\mu)$$
.

In order not to exceed the Galileo maximum hardware yaw rate *R* (~0.20 deg/s) during noon and midnight turns for small  $|\beta| < 2$  deg, when *E* is within the eclipsing limits of

 $|E| < 15 \deg for \ midnight$ , or  $|E-180| < 15 \deg for \ noon \ turns$ , (4)  $S_v$  in Eq. 3 is replaced with the model  $S_{hv}$ , evaluated as

$$S_{hv} = (\sin \beta_v \Gamma + S_v)/2 + (\sin \beta_v \Gamma - S_v) \cos(\pi/S_x)/\sin \beta_x)/2, \tag{5}$$

where  $\Gamma$  is the sign of  $S_y$  (or of  $-\beta$ ) at the start of the turn, the evaluated  $S_{hy}$  is then stored the subroutine internal variable *BETAE*. Then, the eclipsing model yaw angle  $\psi_s$ , during the Galileo eclipsing noon or night turn period, is

$$\psi_s = ATAN2(S_{hv}, S_x), \tag{6}$$

and used to reorient the body-x vector. In all other times when  $|\beta| \ge 2 \deg$ , or for  $|\beta| < 2 \deg$  with |E|, or  $|E-180| \ge 15$  deg, the nominal yaw  $\psi_n$  of Eq. 3 is used (i.e. the input body-x vector orientation is not changed). From (5), one can see that for the eclipsing limits  $\mu = \pm 15$  or  $(180 \pm 15)$  deg, the value of the of the fraction in the cos term in (5) becomes equals to  $-\cos\beta$ , i.e.,  $\sim -1$  and then the cos term also becomes nearly equal to -1. This nearly cancels both sin  $\beta_y$  terms in (5) and  $S_{hy} \sim S_y$  and  $\psi_s \sim \psi_n$ . At turn middles, when  $\mu = 0$  or 180 deg. the fraction of the cos term becomes 0 and the cos term is equal to 1, which then cancels both (very small)  $S_y$  terms of (5), and depending on the sign  $\Gamma$ ,  $S_{hy} \sim \pm \sin \beta_y$ , i.e.  $\beta = \beta_y = 2$  deg is used for the model yaw angle  $\psi_{0}$ , ensuring that the maximum hardware yaw rate is not exceeded. It takes about 70 min for a Galileo satellite to complete an eclipsing (noon or midnight) turn period, specified by limits of (4). During the first 15 min and the last 15 min of a Galileo model eclipse turn, the nominal and the model yaw angles agree within a few deg, even for a very small  $\beta$  angle (~ 0 deg). For  $|\beta|$  between 1.5 and 2 deg, the model  $\psi_s$  and the nominal  $\psi_n$  are nearly the same (within about 5 deg) during the whole turn. The  $\beta_{y} = 2$  deg limit is hard-coded in the *BETA0* internal variable, which is also used for Beidou. Note that setting BETA0 = 0 disables all the Galileo as well as Beidou IGEO/MEO satellite eclipsing'. In Eq. 5 the sign  $\Gamma$  can be replaced with the opposite sign of the current  $\beta$ , provided that  $\beta$  does not change the sign during the turn interval. If there is a  $\beta$  sign change, which is a rather rare occurrence, but it can happen in particular for the long Galileo eclipse turns, then using the current  $\beta$  sign to determine the sign  $\Gamma$  can cause large errors and discontinuities in the model yaw angle  $\psi_s$ , in particular within  $\pm 20$  min of the mid turn time. This is why, for extremely small  $\beta$  (within  $\pm 0.07$  deg), for which a sign change is possible,  $\beta$  at the turn start is saved in the newly introduced call parameter *BETAINI*, and it is used to determine the sign  $\Gamma$  in (5) for small Sun angles  $|\beta| \le 0.07$  deg.

# Initial turn $\beta$ (for GPS, GLONASS and Galileo), and GPS Block IIF and IIA anomalous noon turn directions for small negative and positive $\beta$ , respectively (source-code changes/update "Feb 27, 2017")

The change of  $\beta$  sign during an eclipsing turn, if unaccounted for, can cause significant errors in GPS, GLONASS and Galileo Eclipsing. In particular, due to the long turn duration of 70 min, the Galileo turns are most likely to experience the rare  $\beta$  sign change. Note that the Beidou eclipsing is not affected by any  $\beta$  sign change, as no turns are employed when  $|\beta| \le 2$  deg. To eliminate this problem, a call parameter array *BETAINI* (corresponding to each *IPRN*), which should be externally initilized to zeros, has been introduced. If a GPS, GLONASS or Galileo satellite (during a forward processing (*IDIR* = 1)) has  $|\beta| \le \beta$ 

0.07 deg and the *BETAINI(IPRN)* = 0, then *BETA(IPRN)* is set to  $\beta$ . This ensures that an initial  $\beta$  value is saved in *BETAINI* only when a  $\beta$  sign change is possible during a turn. Note that for  $|\beta| > 0.07$  deg the sign cannot change during the turn (i.e., for up to the next 70 min), also note that a single *BETAINI* value for each satellite is sufficient, since the next  $\beta$  sign change during an eclipsing turn cannot happen for about six months.

In all subsequent GPS, GLONASS satellite (*IPRN*) eclipse processing, with  $|\beta| \le 0.07$  deg and *BETAINI(IPRN)*  $\ne 0$ , *BETAINI(IPRN)* is used instead of the actual  $\beta$ . This ensures the correct noon turn directions for all GPS and GLONASS satellites as well as for night turns of GPS Block IIR/IIRM and IIF ones, since the difference between the start yaw and end Block IIF yaw cannot exceed  $\pm 180$  deg. For Galileo satellites with  $|\beta| \le 0.07$  and *BETAINI(IPRN)*  $\ne 0$ , the *BETAINI(IPRN)* is used for the sign  $\Gamma$  assignment, along with in the actual  $\beta$  in  $S_y$  and  $S_x$  of (5). When a Galileo satellite  $|\beta| > 0.07$ , then  $\beta$  is used both for the sign  $\Gamma$  determination as well as the rest of (5).

Dilssner et al. (2011) has observed anomalous noon turn directions for GPS Block IIF satellites with small negative  $\beta$  between 0 and about -0.9 deg. These IIF anomalous noon turns can be fully accounted for by a positive yaw bias of 0.9 deg, consequently they were also implemented into the December 2013 version of the *eclips* subroutine with the  $\beta$  limit of [-0.9, 0 deg) for the anomalous Block IIF noon turns. Recently, Kuang et al (2016) has used reverse PPP's during 1 year period to investigate all the Block IIF noon turns with small  $\beta$  ranging from -1.5 to +1 deg. With only 2 exceptions, both for PRN 25 (SVN 62,  $\beta = -0.8$  and +0.5 deg), all the Block IIF noon turns within the  $\beta$ -interval of [-0.7, 0 deg), as expected, had the anomalous (wrong) turn directions. All the Block IIF noon turns with  $\beta < -0.7$  deg had the correct direction, with only a single exception of the PRN25 with  $\beta$  = -0.8 deg, otherwise, this satellite also was turning as the rest of the Block IIF satellites with small negative  $\beta$ . Considering the above findings, the Block IIF yaw bias has been changed to -0.7 deg and also used as the limit of anomalous noon turns with small negative  $\beta$  ( $|\beta| \le 0.7$  deg). If required, the limit of anomalous Block IIF noon turns can be changed by changing the Block IIF yaw bias (YBIAS). Alternatively, the Block IIF anomalous noon turns can be disabled by using YBIAS = 0 deg. During the one year period, Kuang et al (2016) has also detected one anomalous Block IIF night turn, which was due to the fact that the model turn, required for the shadow crossing, exceeded 180 deg, which resulted in a wrong turn direction. This is why it was recommended that the Block IIF yaw angle difference between the shadow start and end, used for shadow yaw rate computation, be tested and if it exceeds  $\pm$  180 deg, a 360 deg complement resulting in a turn of less than 180 deg is used (Kuang et al 2016). Note that this test is not used here as it is not necessary, since here the use of *BETAINI(IPRN*) for  $|\beta| \le 0.07$  deg, ensures that the yaw difference and as well as the turn are always less than 180 deg.

The Block IIA yaw bias of 0.5 deg should also cause wrong direction noon turns for small positive  $\beta \leq 0.5$  deg, some such anomalous Block IIA noon turns have already been observed. This is why the Block IIA anomalous noon turns for  $\beta$  of (0, 0.5 deg] have also been implemented in this version of the *eclips* subroutine. The anomalous Block IIA noon turns can be disabled, as indicated in the source-code updates (labelled by "*Feb 27, 2017*"), namely by changing the IF statement of "*IF(IPRN.LE.32.AND.BLK(IPRN).EQ.6.AND.*". Note that since January 2016, when the last Block IIA satellite (PRN 32/SVN23) has been decommissioned, there are no Block IIA satellites operational.

# Testing

The Feb 2017 *eclips* version has been extensively tested using all the GPS Block types, including the anomalous IIF and IIA noon turns, as well GLONASS and Galileo eclipsing, all with and without  $\beta$  sign changes. Testing of Beidou eclipsing has also included all the three types of Beidou satellites, i.e. MEO, IGEO as well as GEO. Note that  $\beta$  sign changes do not have any effects in Beidou eclipsing. Analyses of Galileo and Beidou PPP phase residuals along with the corresponding MGEX (GRM and GBM) clock solutions have indicated that the above eclipsing models have been properly implemented and also that the implemented eclipsing models are indeed used by the Galileo and Beidou satellites.

# References

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