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ORIGINAL ARTICLE

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Efficient interpolations to GPS orbits for precise wide area applications

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Abstract For precise real time or near real time differential GPS positioning in a wide or global area, precise GPS orbits or, alternatively, precise orbital corrections with respect to a reference orbit, such as GPS broadcast ephemerides, must be used. This work tests orbit interpolation methods, in order to represent the GPS orbits and orbital corrections accurately and efficiently for these and other GPS applications. For precise GPS orbits given in the SP3 format at the 15 min interval, numerical tests were conducted using Lagrange and Chebyshev as well as trigonometric polynomial functions. The results have demonstrated that the 19- or 20-term trigonometric function is apparently the most efficient interpolator for a 12 h GPS orbital arc, achieving 1 cm level 3D interpolation accuracy that can meet the requirements of most precise applications. The test results also demonstrated that the 9-term trigonometric function always yields optimal interpolation for a 2 h GPS orbit arc, in terms of interpolation errors, compared to the results when using a different number of terms for the same function or one of the other tested polynomial functions. This is evident from the minimal performance degradation when using the 9-term trigonometric function to interpolate near or at the end of a data interval. By limiting interpolation to the center 15 min to 1.5 h of a 2 h orbit arc, thereby eliminating the need to interpolate near the ends of that interval, users can opt for more terms (11 and 13) or different interpolators to further improve interpolation accuracy. When interpolating the orbital corrections with respect to the GPS broadcast ephemeris, all the tested interpolation functions of 3- to 9-term yield the same suitably accurate results. Therefore, a 3- to 5-term trigonometric function is arguably sufficiently accurate and more efficient for GPS orbital correction messaging in wide area and real time positioning.

Introduction

The International GPS Service (IGS) collects GPS data from its worldwide network for a wide range of scientific and engineering applications and studies. The IGS uses these data sets to generate three types of precise GPS satellite ephemerides: ultra-rapid, rapid and final ephemerides, with claimed 3D accuracy of 25 cm, 5 cm and less than 5 cm, respectively. While the ultra-rapid ephemeris is available for real time applications, the rapid and final ephemerides, which are suitable for high precision applications, have a latency of 1 day and 2 weeks, respectively (Spofford and Remondi 1999; http://igscb.jpl.nasa.gov). In addition, the Jet Propulsion Laboratory (JPL) uses the data from NASA's global differential GPS network to produce Real Time GIPSY (RTG) GPS orbits every 15 minutes which can be made available to users through prior arrangements. The IGS and JPL precise ephemerides have a tabular interval of 15 min and 5 min, respectively. The positions and velocities at any time instant between the 15 min or 5 min entries are typically obtained by interpolation but may come from some other method. There are a variety of mathematical functions and software tools available for interpolation. Here, the primary requirement that the interpolation of the ephemeris should not result in significant degradation of the orbit accuracy, which, in turn, can cause loss of precision in the derived GPS solutions, was used to judge the efficiency of different interpolation tools and strategies. For instance, an interpolation scheme is not considered good enough if the interpolation errors are greater than the ephemeris uncertainty. Fortunately, by carefully choosing number of terms of an interpolation function, one can effectively minimize the loss of accuracy in the interpolation process leaving several viable possibilities.

In the past, the typical technique used by the DMA (Malys and Ortiz 1989), NGS (Remondi 1991), JPL and other organizations is a Lagrange interpolator with orders varying from 8th to 11th (9- to 12-term). It was generally found that an 8th order Lagrange interpolation using 15 min data can result in an interpolation error at 1 cm level with an interpolation RMS value of millimeters at the center data points (Remondi 1991; Schueler 1998). The recent article by Schenewerk (2003) gave a brief review of basic GPS orbit interpolation strategies pointing out the 5 cm benchmark in the evaluation of an interpolation method for GPS orbit. However, the results given in the article showed that the maximum interpolation errors from the tested polynomial and trigonometric functions for the earth inertial coordinates reached 39 cm and 8 cm at best. While these errors occurred when interpolating near the end point of an ephemeris or extrapolating an IGS ephemeris from its end point at 23:45 through then of the day, this raised a concern of whether the proposed interpolation methods can really satisfy the 5 cm benchmark or better precision.

Previously, the authors had tested four functions for interpolating precise GPS ephemerides (Feng et al. 2004) seeking suitably accurate methods and schemes to fulfill the requirements for GPS orbits in near-real time and real time applications, such as wide area differential GPS positioning, more efficiently. That paper concluded that the 9-term trigonometric function is optimal to represent 2 h GPS orbits arcs and orbital corrections with regards to accuracy and efficiency. But this same function gave unacceptably large maximum 3D interpolation errors for precise GPS orbits and RTG orbits. These errors, like those from Schenewerk (2003), arise from interpolation errors at the ends of each data interval.

This paper presents results from further studies into the effects of the interpolation, providing a more systematic understanding of the GPS ephemeris interpolation issues. We first discuss the differences and similarities of different interpolators and strategies being used for GPS orbits. Next, we examine numerically the effects of interpolating near the end points of an ephemeris with different numbers of terms for the Chebyshev and trigonometric polynomials. Tests are also conducted to evaluate the performance of interpolation when applied to the coordinate differences between the precise and broadcast GPS ephemerides, addressing the needs for real time GPS positioning applications.

Interpolators and interpolation strategies

There are many interpolation methods suitable for GPS orbit representation. Neta et al. (1996) discussed and compared several methods for polynomial interpolation of GPS orbits, including Lagrange, Newton's divided difference, Difference tables, Cubic Spline, Chebyshev and trigonometric polynomials. In our previous study (Feng et al. 2004), four interpolators, Lagrange, Neville, Chebyshev and trigonometric algorithms were examined. The Lagrange's and Neville's algorithms gave exactly the same results (Feng et al. 2004), of necessity; therefore, in this study, we only chose the three interpolators: Lagrange, Chebyshev and trigonometric methods, for examination. First, let us summarize some straightforward observations about these three interpolators:

- Lagrange interpolator requires the terms of the polynomial to be equal to the number of the data points, while Chebyshev interpolator has no such requirement. Given n data points on an interval, an n-term Lagrange polynomial is needed to interpolate these data points uniquely. Numerical results have shown that Chebyshev interpolators can give results identical to the n-term Lagrange polynomial when interpolating the same n data points. Although the Lagrange and Chebyshev polynomials take different variable bases, they will result in the same polynomial as long as the data points are equally spaced and the number of terms used in the polynomial is equal to the number of data points. Trigonometric interpolators, however, use a totally different polynomial function and yield different results.
- However, a different set of n data points on the same interval would yield a different polynomial. In other words, on a given interval, some choices of n data

points would result in polynomials that can provide interpolation results better than others. In the IGS GPS orbit case, the data points are evenly spaced throughout of the interval of interest, the choice of using all the n points seems the best in terms of interpolation accuracy, unless one chooses to under sample the data points for other reasons.
– Chebyshev and trigonometric interpolators allow the use of the n-term polynomials to fit more than n data points for better interpolation results, while Lagrange interpolation cannot. The least square estimation is normally used to fit coefficients of the polynomials or functions.

One common problem of the above interpolators is the fact that the error between the interpolating polynomial and the data being interpolated grows rapidly near the end-points of the interval. This is especially the case when the n-term polynomial is used to interpolate n evenly spaced data points. To address this problem, the current strategy is to use only the central subinterval of the Lagrange polynomial generated over a given interval. The polynomial is produced successively, from points 1 to n, and 2 to $n+1$,..., so called "Walk-Along interpolation". The method may achieve the highest level of accuracy. The numerical accuracy has been verified to the 1 cm level for the GPS orbits (Remondi 1991). But, the disadvantage of the method is that it is computationally expensive. From the practical point of view, as long as the accuracy requirements can be met, more efficient interpolation strategies may be desirable. For these reasons, we compared the interpolators and interpolation strategies as follows:

– Scheme I: For a given interval with n data points, the n-term polynomials are used to interpolate to end points. All the three interpolators are tested to determine the optimal number of terms (orders) and find the best function, where the terms range from 5 to 17.
– Scheme II: Using Scheme I results, examine the performance degradation towards the end points and determine how far from the end points one must stop interpolation to meet the accuracy requirements.
– Scheme III: Again using Scheme I results, examine the accuracy achievable over the central subinterval from all three interpolators.
– Scheme IV: For a given interval of one GPS orbit (about 720 minutes, 48 subintervals, 49 data points), examine the performance of Chebyshev and trigonometric polynomials of different terms, and seek the most efficient and accurate polynomials.

With selected interpolators, three interpolation data sources will be used: precise JPL post-fit orbits, JPL RTG orbits and orbital corrections with respect to GPS broadcast orbits. The results are presented in the following section.

Experiments and interpolation results

The standard IGS orbits are given at a 15 min data interval. Therefore, the basic test procedures include interpolating a standard source ephemeris at 15 min data interval and producing satellite coordinates at shorter intervals, such as 30 s or 5 min. The interpolator outputs are then compared to a reference ephemeris also with 30 s or 5 min tabular intervals. The difference between the interpolated coordinates and the reference coordinates at all data points over the whole interval or orbit are used to statistically evaluate the accuracy of the interpolator.

Scheme I test results

The first round of tests is to study the interpolation accuracy including the endpoints as defined in *Scheme I*. We first recreated and extended the results from Schenewerk (2003). The test data, including both source and reference ephemerides were taken from the website (http://www.ngs.noaa.gov/gps-toolbox) and we followed similar analysis procedures. In this analysis, the data points used were equal to the number of coefficients in all cases. Table 1 summarizes the results from the Earth-Centered and Earth-Fixed (ECEF) interpolating and reference ephemeris tests for PRN 01. For each interpolator, two columns are given: the standard deviation (SD) and the maximum difference of the interpolated minus reference coordinates within the time span of 00:00 to 24:00 of the entire day. From Table 1, it is evident that all types of the tested functions provide the interpolation results with standard deviations at the millimeter level and maximum deviations under the 5 cm benchmark if 9- to 13-term are used. Function variants with fewer than 9 and more than 13 terms led to degradation of the original orbit information. The Lagrange and Chebyshev functions gave the same results while the 9-term trigonometric function performed best. This implies that the 9-term

Table 1 The standard and maximum deviations of the functions representing the ephemeris' ECEF coordinates for PRN 01 versus the number of polynomial terms used in the function (unit: cm)

Terms	Lagrange		Chebyshev		Trigonometric	
	SD	Max	SD	Max	SD	Max
5	913.7	5595.4	913.7	5595.4	423.9	2506.0
7	12.5	126.1	12.5	126.1	0.6	6.3
9	**0.2**	**3.4**	**0.2**	**3.4**	**0.1**	**1.3**
11	0.2	2.5	0.2	2.5	0.2	2.4
13	0.3	4.9	0.3	4.9	0.3	4.6
15	0.8	11.4	0.8	11.4	0.8	10.4
17	2.6	32.7	2.6	32.7	0.3	3.2

trigonometric function could be an ideal representation method of 2 h GPS orbits in an ECEF coordinate system.

A basic concern is whether the performance of the interpolators varies from one source ephemeris to another, therefore two other GPS ephemerides were tested in our studies. One was the JPL precise post-fitted ephemerides (JPL ephemerides) with a 30 s interval, and the other was the JPL RTG ephemerides with a 5 min interval. In this test, the control ephemeris was the original ephemeris while the source ephemeris for interpolation was the original ephemeris decimated to a 15 min interval.

A sample summary of standard and maximum deviations for PRN 01 from the interpolation of the JPL post-fit precise ephemeris for the day 1 January 2004, GPS Time from 00:00:00 through to 23:59:30 is listed in Table 2. The trigonometric polynomial interpolation again performed better than other two interpolators. Similarly, the 9-term trigonometric polynomial achieved the best interpolation SD of approximately 1 cm. The 9-term Lagrange and Chebyshev interpolators also perform best compared to results from those interpolators with greater or fewer numbers of terms, but are slightly worse than those from the trigonometric polynomial. Therefore, the nine-term trigonometric polynomial appears to consistently give the necessary accuracy and provide the most accurate results for a 2 h GPS orbit arc (eight 15 min epochs). The SDs of the nine-term interpolations for all satellites available during the day are illustrated in Fig. 1. Because Lagrange and Chebyshev interpolators produced identical results so only the Lagrange and trigonometric results are plotted. As clearly shown in Fig. 1, all the satellites have the interpolation standard deviations of less than 1.6 cm and 0.7 cm for Lagrange and trigonometric interpolators, respectively.

Next, interpolation of the RTG ephemeris for the same day, 1 January 2004, was evaluated. Here, the control, i.e., original ephemeris, had a 5 min interval. Table 4 gives the SD and maximum deviation of all the interpolators for PRN 01 versus the number of terms of interpolating functions. Figure 2 shows the SDs of the 9-term interpolations for all satellites available. Again, the Lagrange and Chebyshev interpolators gave identical results so only the Lagrange and trigonometric results are shown. While most satellites had interpolation SDs of around 2 cm, SDs of 5–10 cm were seen for the PRN 04, 15, 17 and 24. The standard and maximum deviations for PRN 04 are summarized in Table 4. Note that the maximum deviation reaches 114 cm for the 9-term trigonometric interpolator. Examination of these orbits revealed that these four satellites were moving through the shadow of the Earth, i.e., in an eclipse period, which caused the source ephemeris to be less accurate and, therefore, resulting in larger interpolating errors. Figure 3 illustrates the RMS value for difference between JPL precise and RTG orbit solutions, showing the large orbit errors of PRN 04 comparing others.

Table 2 The standard and maximum deviations of the interpolators for PRN 01, given against the terms of interpolating functions for the post-fit precise JPL orbits (unit: cm)

Terms	Lagrange		Chebyshev		Trigonometric	
	SD	Max	SD	Max	SD	Max
5	1678.1	5811.8	1678.1	5811.8	783.8	2747.4
7	38.7	141.2	38.7	141.2	1.9	8.3
9	**1.1**	**7.8**	**1.1**	**7.8**	**0.6**	**4.3**
11	1.4	10.3	1.4	10.3	1.3	10.1
13	3.8	29.2	3.8	29.2	3.5	27.5
15	10.9	89.0	10.9	89.0	9.7	80.1
17	32.5	276.2	32.5	276.1	5.8	48.9

Table 3 The standard and maximum deviations of the interpolators for PRN 01, given against the number of orders of the interpolating functions for the JPL RTG ephemerides (unit: cm)

Terms	Lagrange		Chebyshev		Trigonometric	
	SD	Max	SD	Max	SD	Max
5	1956	5800.1	1956.9	5800	914.2	2741.5
7	40.3	143.6	40.3	143.6	2.3	10.5
9	**2.1**	**17.1**	**2.1**	**17.1**	**1.8**	**16.0**
11	4.2	38.3	4.2	38.3	4.2	37.0
13	11.0	106.5	11.0	106.5	10.3	99.7
15	31.2	324.6	31.2	324.6	28.0	292.1
17	92.9	1034.6	92.9	1034.	18.3	177.2

Scheme II and III test results

As stated, the test *Schemes II* and *III* were designed to examine the performance degradation which can occur near the ends of an interpolation interval, identify interpolation limits within the interval which meet the accuracy requirements and determine the accuracy achievable over the central subinterval or subintervals. Figure 4a–e shows the interpolation errors of Lagrange and trigonometric polynomials over the orbit arc of 1–3 h, using 5–13 polynomial terms. From these figures, following is clearly observed:

- Interpolation errors increased significantly in the first and last 15 min intervals.
- The trigonometric polynomial performed better than the Lagrange polynomial in lower polynomial order cases.
- Both polynomials achieved the best performance with the 9-term polynomial in the sense of minimal inter-

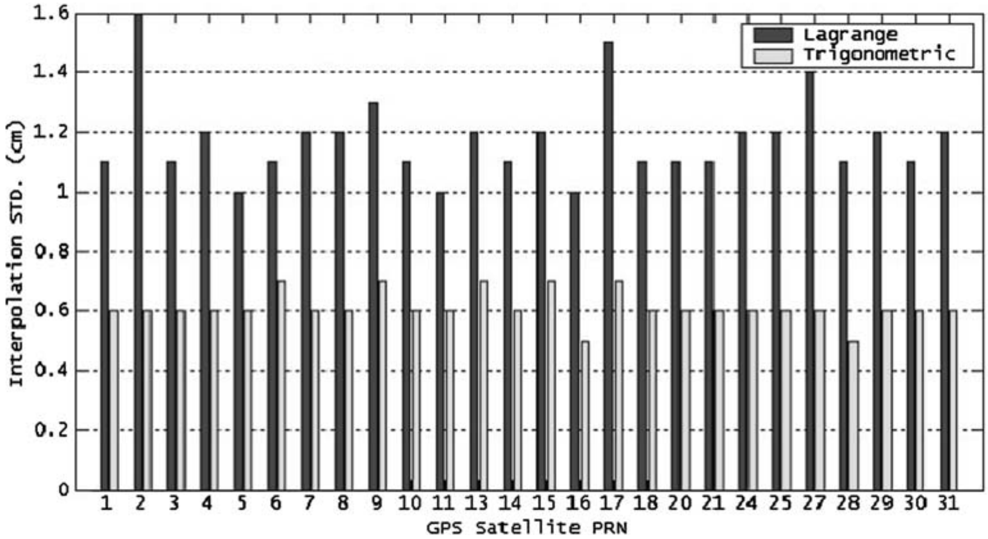


Fig. 1 Comparison of SDs between the nine-term Lagrange and trigonometric interpolation functions for the post-fitted GPS orbits of the satellites

Table 4 The standard and maximum deviations of the interpolators for PRN 04 orbit, against the number of terms used in interpolating real-time RTG orbits (unit: cm)

Terms	Lagrange		Chebyshev		Trigonometric	
	SD	Max	SD	Max	SD	Max
5	1904.8	5861.8	1904.8	5861.8	889.3	2678.3
7	39.5	169.5	39.5	169.5	5.9	59.9
9	**10.2**	**114.4**	**10.2**	**114.4**	**10.0**	**113.6**
11	22.5	281.3	22.5	281.3	21.7	269.0
13	57.2	752.5	57.2	752.5	53.7	697.4
15	159.8	2141.3	159.8	2141.3	143.7	1912.2
17	472.7	6617.4	472.7	6617.4	93.7	1442.0

polation errors at the end intervals, although the trigonometric function showed slightly better results. This confirms the results from *Scheme I* tests.

– Limiting interpolation to the center subinterval provided the highest accuracy, typically, at the 1 cm SD level. But, if an interpolation error of approximately 1.5–2 cm is acceptable, the limits can be extended to include all six center intervals (1.5 h) with 9- to 13-term polynomial Lagrange or trigonometric function.
– Overall, choosing the 9-term trigonometric function to interpolate either 2 h (including two end intervals) or the center 1.5 h orbit (excluding two end intervals) would apparently be the most efficient and safest interpolation scheme for GPS orbits. Including a 30 min overlap from one subinterval to the next gave standard deviations less than 1.5 cm. An orbit interpolation accuracy of only 5 cm or less was possible when overlapping epochs were not permitted.

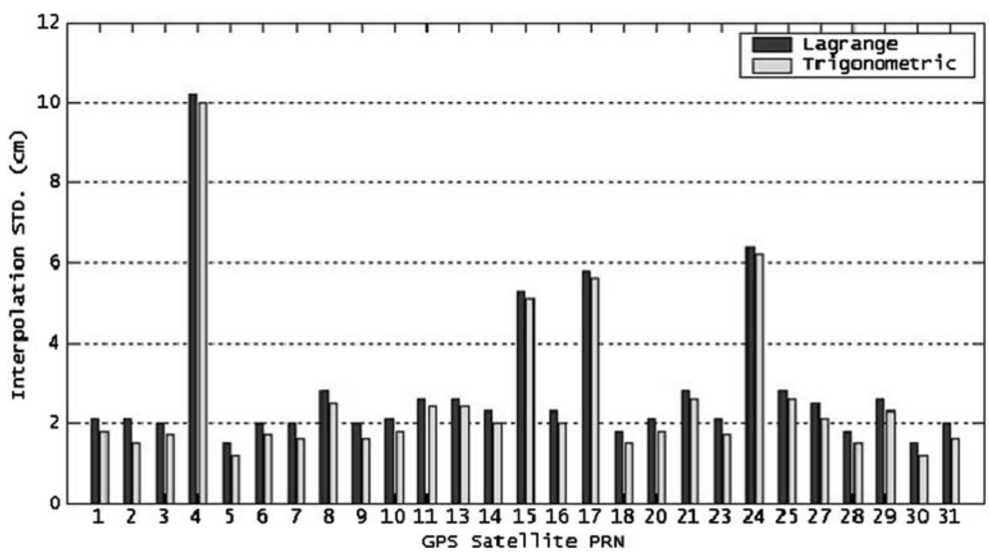


Fig. 2 A comparison of SDs when using the nine-term Lagrange and trigonometric functions to interpolate the RTG ephemerides

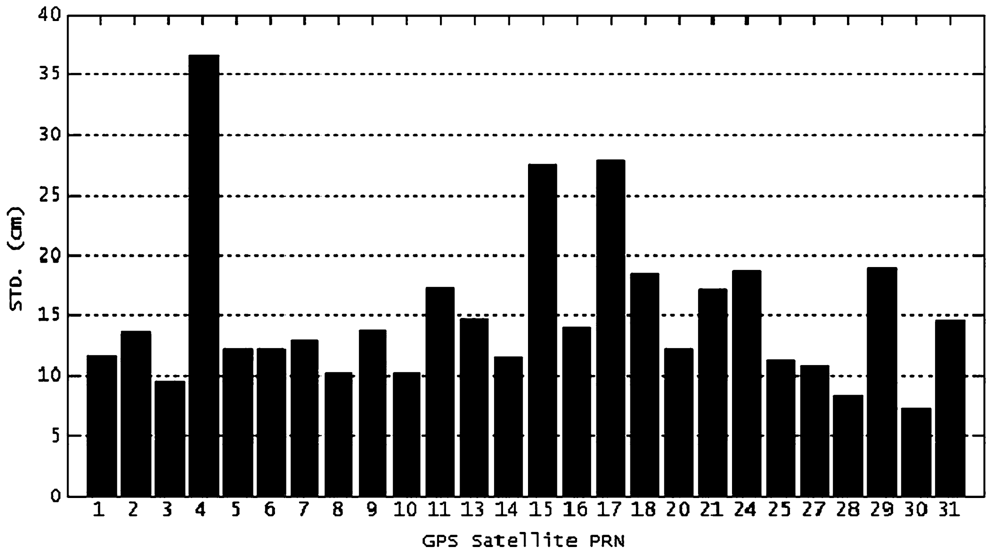


Fig. 3 RMS values of JPL RTG orbits with respect to the JPL precise orbits for all the satellites

Scheme IV test results

Scheme IV examined the performance of Chebyshev and trigonometric polynomials interpolators with various numbers of terms using one half day GPS ephemeris (720 minutes, 48 subintervals, 49 data points) in order to identify the most efficient and accurate polynomials. Figure 5a–h shows the 3D orbit interpolation errors across the whole interval produced by the Chebyshev and trigonometric polynomial interpolators with 19–26 terms. The results can be summarized as follows:

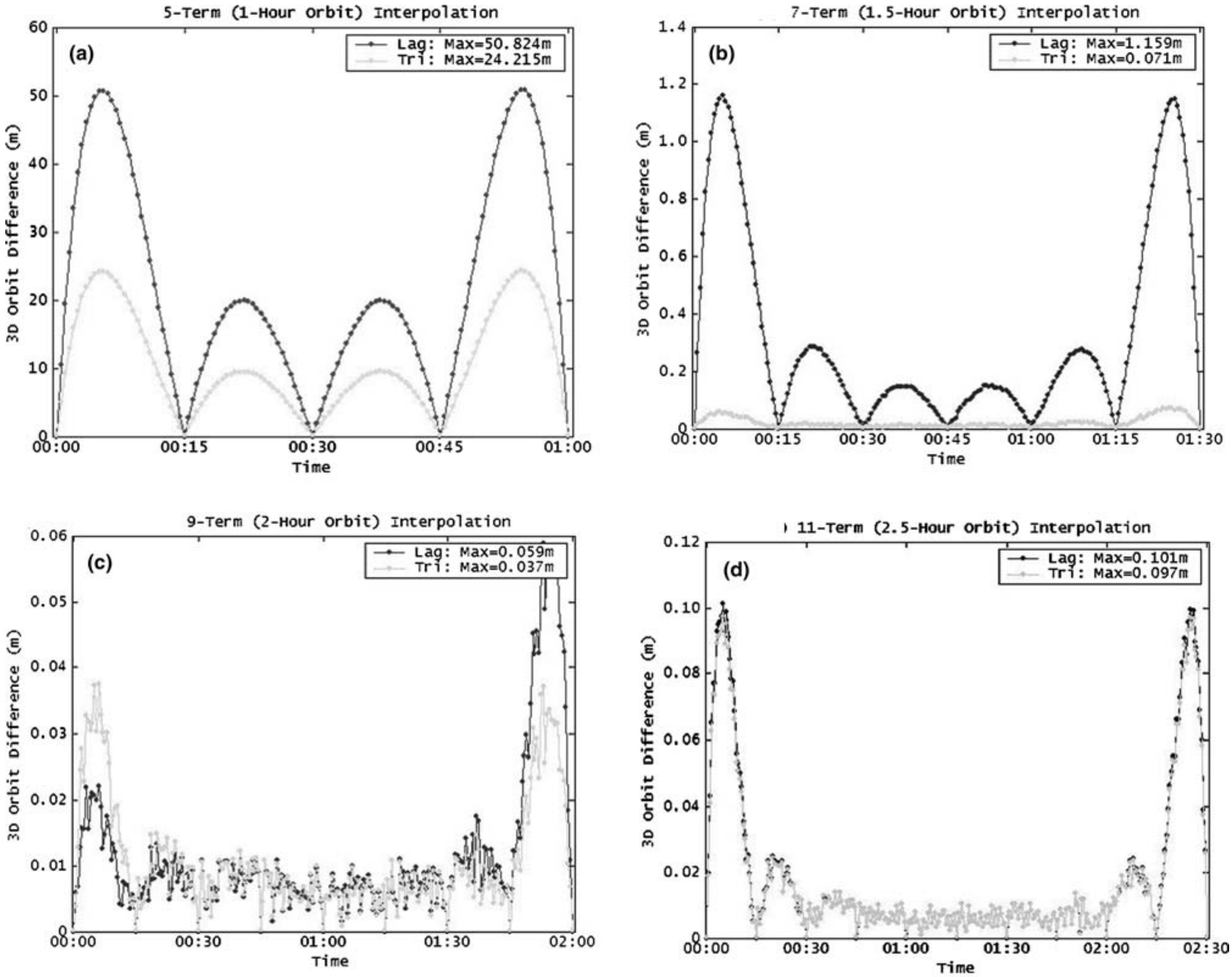
- The edge effects of interpolation with the trigonometric polynomial at the ends of an interval could be removed by carefully choosing the number of polynomial terms (or orders). With Chebyshev polynomial interpolation, the error could be reduced to a few centimeters. Therefore, both the trigonometric and Chebyshev polynomials can efficiently and accurately represent a 12 h GPS orbit.
- The 19- or 20-term trigonometric polynomials performed the best in these tests with the 3D maximum deviation of 1.1 cm and RMS of 0.6 cm. This result is comparable to the 2 h, limited interpolation explored in *Scheme II* and *III*.
- The 21- to 22-term Chebyshev polynomials can yield good results as well giving a 3D maximum deviation of 2.2 cm and RMS of 0.6 cm.

Interpolating the orbital corrections

For real time wide area positioning, the precise orbit information should be used to correct the GPS broadcast ephemerides. Currently in the wide area augmentation system (WAAS), GPS orbital positional and velocity collections are updated at 15 min interval, while the orbital corrections are broadcast every 2 min. The IGS ephemerides products are moving toward real time by producing a predicted ephemeris every 6 h. At some IGS data processing centers, the GPS predicted ephemerides are updated hourly. Therefore, orbit corrections can be created from these predicted ephemerides and updated at enhanced frequency as well. In practice, orbit correction updates must consider the update of GPS broadcast ephemerides, typically at the interval of 2 h, as well. Applying the above interpolation methods to the coordinate differences can produce the correction functions, at variable intervals, for instance 30 min to 2 h, allowing to cover a longer orbital period.

Figure 6 plots the x, y, z differences of a precise GPS ephemeris with respect to the GPS broadcast ephemeris over 24 h. Note the discontinuities. This is often caused by the introduction of a new set of GPS broadcast ephemeris parameters. Therefore, the corrections must be recomputed from time to time in synchronization with any broadcast ephemeris updates. Table 5 gives the standard and maximum deviations from the interpolators for PRN 01 orbital corrections. It is clearly seen that all the interpolators with the terms of three to nine give almost the same results.

Because the orbit correction message can be updated frequently, for instance every 15 min in the WAAS message, a four-term polynomial may be a good choice to cover the central 15 min and additional 15 min to the later end point with centimeter level interpolation accuracy. Comparisons to the WAAS position and velocity correction representations shows that four-term



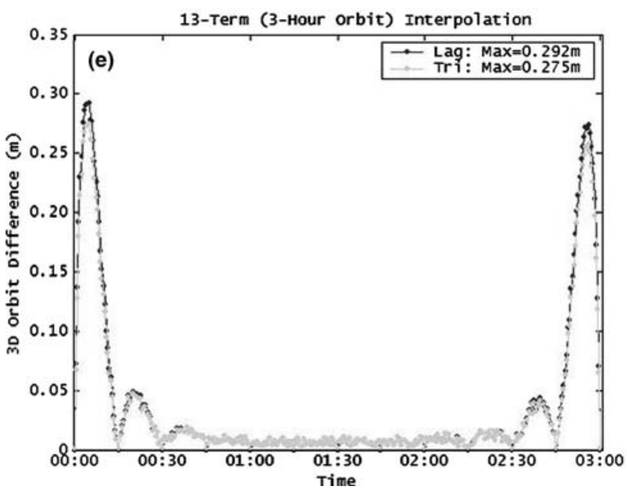
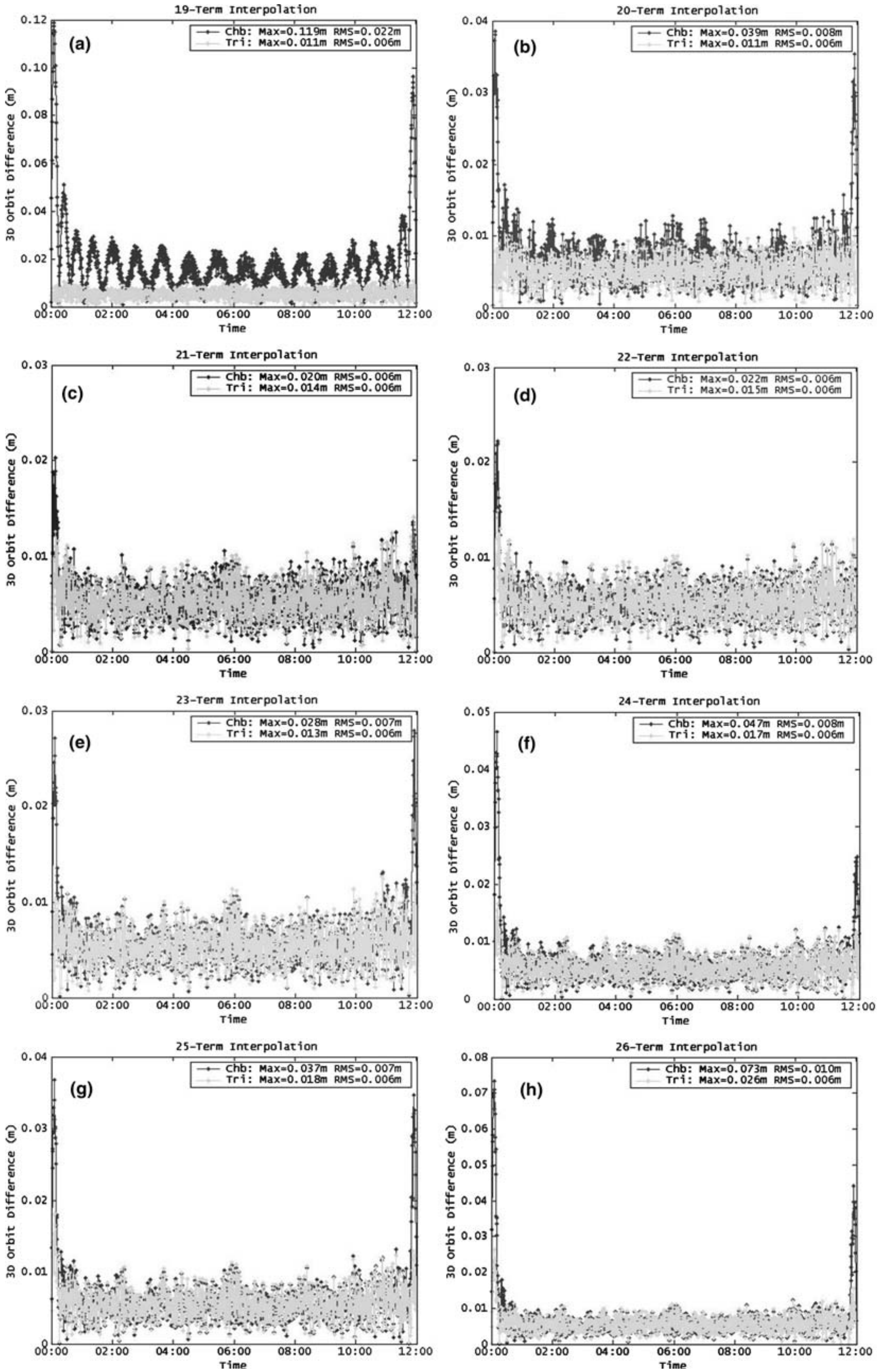


Fig. 4 **a**–**e** Illustration of the interpolation errors of Lagrange and trigonometric polynomials over the orbit intervals from 1 h to 3 h, where the polynomial terms range from 5 to 13 accordingly

polynomials can efficiently control error growth beyond the 15 min interval of the limited time span of the data messages for orbit corrections.

Table 6 provides a summary of the trigonometric interpolation results from different test schemes for the JPL precise orbits in terms of interpolation accuracy



19-Term Interpolation
(a)
Chb: Max=0.119m RMS=0.022m
Tri: Max=0.011m RMS=0.006m
20-Term Interpolation
(b)
Chb: Max=0.039m RMS=0.008m
Tri: Max=0.011m RMS=0.006m
21-Term Interpolation
(c)
Chb: Max=0.020m RMS=0.006m
Tri: Max=0.014m RMS=0.006m
22-Term Interpolation
(d)
Chb: Max=0.022m RMS=0.006m
Tri: Max=0.015m RMS=0.006m
23-Term Interpolation
(e)
Chb: Max=0.028m RMS=0.007m
Tri: Max=0.013m RMS=0.006m
24-Term Interpolation
(f)
Chb: Max=0.047m RMS=0.008m
Tri: Max=0.017m RMS=0.006m
25-Term Interpolation
(g)
Chb: Max=0.037m RMS=0.007m
Tri: Max=0.018m RMS=0.006m
26-Term Interpolation
(h)
Chb: Max=0.073m RMS=0.010m
Tri: Max=0.026m RMS=0.006m
3D Orbit Difference (m)
Time

◄

Fig. 5 a–h Illustration of the 3D orbit interpolation errors obtained from Chebyshev and trigonometric polynomials of 19 to 26-term in interpolating one GPS orbit period

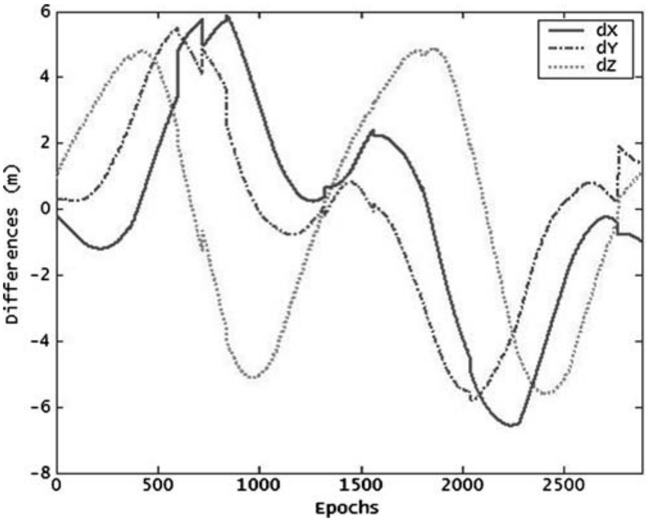


Fig. 6 Coordinate differences of a precise GPS orbit with respect to the GPS broadcast orbit over 24 h

Table 5 The standard and maximum deviations of the interpolators for PRN 01 orbital corrections (unit: cm)

Terms	Lagrange		Chebyshev		Trigonometric	
	SD	Max	SD	Max	SD	Max
3	0.6	3.1	0.6	3.1	0.6	3.1
5	0.6	2.9	0.6	2.9	0.6	2.8
7	0.6	2.9	0.6	2.9	0.6	2.9
9	0.6	3.0	0.6	3.0	0.6	3.0
11	0.6	3.5	0.6	3.5	0.6	3.5
13	0.7	5.4	0.7	5.4	0.7	5.1
15	1.6	13.1	1.6	13.1	1.4	11.9

and efficiency. An interpolation strategy is more efficient if the message size needed to represent an orbit interval is smaller than the IGS SP3 data size for the same orbit interval. As seen, choosing to interpolate the central 1.5 h orbit instead of 15 min interval is much more efficient, with regards to total message size, while retaining almost the same interpolation accuracy. The 20-term trigonometric function provides the smallest message size for 12 h GPS orbits. Representing a 24 h GPS orbit with two 20-term trigonometric function requires less than half the size of the corresponding SP3 ephemeris. Furthermore, providing more terms gives the user a choice in their final interpolation accuracy.

Conclusions

This research has tested methods to efficiently and accurately represent the GPS orbits and orbital corrections with Lagrange, Chebyshev and trigonometric polynomials functions. Both GPS orbital positions and orbit coordinate corrections with respect to a reference orbit can be represented in this manner.

The 19- or 20-term trigonometric interpolator results in the smallest message size for 12 h GPS orbits. For instance, representing a 24 h GPS orbit with two 20-term trigonometric functions requires less than half of the SP3 data size. The user can directly use the derived interpolator polynomial coefficients to compute the satellite position and velocity of any time instant with virtually no loss of accuracy. This suggests that the trigonometric representation of 24 h GPS orbits could be developed as an alternative to the standard SP3 ephemeris format used for IGS services.

Interpolation test results using GPS precise ephemerides demonstrated the 9-term trigonometric function always yielded optimal interpolation, i.e., the greatest accuracy with the smallest interpolation errors, for a 2 h GPS orbit arc compared to the results from the same function with greater or fewer terms or other tested polynomials. This is evident from the minimal performance degradation when using the 9-term trigonometric function to interpolate near the endpoints of a source ephemeris interval. Choosing to interpolate only in the central 15 min to 1.5 h subinterval instead (avoiding interpolating to the endpoints), more terms (9–13) and

Table 6 Comparison of the trigonometric interpolation results from different test schemes for the JPL precise orbits in terms of interpolation accuracy and efficiency

Term	Interval to interpolate	Trigonometric interpolation error (cm)		Data size for 24 h and 29 SVs
		SD	Max	
9	Center 15 min	0.3	1.0	96A + 864B
	Center 1.5 h	0.6	1.2	16A + 144B
	2.0 h	0.6	5.0	12A + 108B
20	12 h	0.6	1.1	2A + 40B
SP3	15 min			96A + 96B

A data length for one time tag, *B* data length for all GPS SV coordinates at each time tag (epoch) different interpolators can be used to further enhance interpolation accuracy.

Interpolation analysis of the orbit corrections for the GPS broadcast messages also shows that using the 3- to 9-term trigonometric function to represent 30 min to 2 h GPS orbital corrections gives optimal results in terms of interpolation accuracy and errors. This presents a viable alternative method to deliver GPS orbital correction messages to users for real time applications.

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